Math 60 9.5 Adding, Subtracting, and Multiplying Radical Expressions

Objectives 1) Add or subtract radical expressions

2) Multiply radical expressions

Recall: Combine like terms
- same bases
- same exponents

① $(3x + 2y - 4) + (5x - 7y - 1)$

$= \underbrace{3x + 5x}_{\text{add coefficients only}} + 2y - 7y - 4 - 1$

$= \boxed{8x - 5y - 5}$

When we want to combine radicals by adding, they must be <u>like radicals</u>.

- Same radicands $\Longleftrightarrow$ same base
- Same index $\Longleftrightarrow$ same exponent

$\sqrt{x} = x^{1/2}$ can be combined only with other $\sqrt{x}$, not with $\sqrt{y}$ or $\sqrt[3]{x}$.

We combine like radicals by adding coefficients.

Add or subtract. Assume all variables are non-negative

② $3\sqrt{5x} + 7\sqrt{5x}$

$= \underbrace{(3+7)}_{\text{add coefficients}}\sqrt{5x}$

$= \boxed{10\sqrt{5x}}$

③ $5\sqrt[3]{11} - 8\sqrt[3]{11} + \sqrt[3]{11}$

$= (5 - 8 + 1)\sqrt[3]{11}$

$= \boxed{-2\sqrt[3]{11}}$

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Assume all variables are non-negative.

(4) $3\sqrt{20} + 8\sqrt{45}$

$= 3\sqrt{4\cdot 5} + 8\sqrt{9\cdot 5}$

index 2 ⇒ want perfect square factors

Not like radicals $\sqrt{20}$ and $\sqrt{45}$ have the same index, but different radicands.

$20 = 4 \cdot 5$ $\quad 45 = 9 \cdot 5$

$= 3\cdot\sqrt{4}\cdot\sqrt{5} + 8\sqrt{9}\cdot\sqrt{5}$ Simplify each radical.

$= 3\cdot 2\cdot\sqrt{5} + 8\cdot 3\cdot\sqrt{5}$ multiply coefficients

$= 6\sqrt{5} + 24\sqrt{5}$ <u>NOW</u> they are like radicals!

$= (6+24)\sqrt{5}$

$= \boxed{30\sqrt{5}}$

(5) $6x\sqrt{12x} - 5\sqrt{3x^3}$ Not like radicals.

$= 6x\cdot\sqrt{4\cdot 3\cdot x} - 5\sqrt{3\cdot x^2\cdot x}$ index 2 ⇒ find perfect square factors.

$= 6x\sqrt{4}\cdot\sqrt{3x} - 5\cdot\sqrt{x^2}\cdot\sqrt{3x}$

$= 6x\cdot 2\cdot\sqrt{3x} - 5\cdot x\cdot\sqrt{3x}$ simplify perfect squares

$= 12x\cdot\sqrt{3x} - 5x\sqrt{3x}$ multiply coefficients

$= (12x-5x)\sqrt{3x}$ <u>Now</u> we have like radicals.

$= \boxed{7x\sqrt{3x}}$

$12x$ and $5x$ are like terms, so these can be subtracted.

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⑥ $2\sqrt{11} + 8\sqrt{6}$

$\boxed{\text{cannot be simplified}}$

$\sqrt{11}$ and $\sqrt{6}$ cannot be simplified because neither 11 nor 6 has a perfect sq. factor.

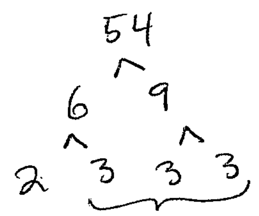
⑦ $\sqrt[3]{-54x^4} + 5x\sqrt[3]{2x} + x\sqrt[3]{16x}$

Assume all variables are non-negative.

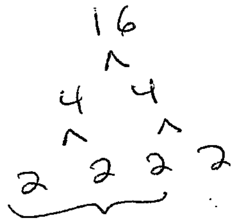
Handwriting alert!

If your handwriting is sloppy, the index 3 of the radical could migrate and become an exponent 3 on the coefficient x.
Avoid this by using extra space, or a multiplication dot.

index 3 ⟹ want perfect cube factors



$3^3 = 27$



$2^3 = 8$

$$= \sqrt[3]{-27 \cdot x^3 \cdot 2x} + 5x\sqrt[3]{2x} + x\sqrt[3]{8 \cdot 2x}$$

Simplify radicals than can be simplified.

$$= \sqrt[3]{-27}\,\sqrt[3]{x^3}\,\sqrt[3]{2x} + 5x\sqrt[3]{2x} + x \cdot \sqrt[3]{8} \cdot \sqrt[3]{2x}$$

$$= -3x \cdot \sqrt[3]{2x} + 5x \cdot \sqrt[3]{2x} + 2x \cdot \sqrt[3]{2x}$$

Now like radicals

$$= (-3x + 5x + 2x) \cdot \sqrt[3]{2x}$$

$$= \boxed{4x\sqrt[3]{2x}}$$

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(8) $2\sqrt{a^2b} - 5a\sqrt{b}$ NOT like radicals

Assume all variables are non-negative.

Simplify $2\sqrt{a^2b}$

index 2 $\Rightarrow$ perfect square

$= 2\sqrt{a^2} \cdot \sqrt{b}$

$= 2a\sqrt{b}$

$= 2a\sqrt{b} - 5a\sqrt{b}$ Now like radicals.

$= (2a - 5a)\sqrt{b}$

$= \boxed{-3a\sqrt{b}}$

Review: Multiply vs Add.

(9) $3\sqrt{5} + 8\sqrt{5}$

$= (3+8)\sqrt{5}$

$= \boxed{11\sqrt{5}}$

combine like radicals

(10) $3\sqrt{5} \cdot 8\sqrt{5}$

$= 3 \cdot 8 \cdot \sqrt{5} \cdot \sqrt{5}$

$= 24 \cdot 5$

$= \boxed{120}$

Multiply and simplify

(11) $\sqrt{6}\,(3 - 2\sqrt{6})$ distribute $\sqrt{6}$.

1st term 2nd term

$= 3 \cdot \sqrt{6} - 2 \cdot \sqrt{6} \cdot \sqrt{6} = 3\sqrt{6} - 2 \cdot 6 = \boxed{3\sqrt{6} - 12}$

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(12) $\sqrt[3]{4}\,(5+\sqrt[3]{2})$ distribute

$= \sqrt[3]{4}\cdot 5 + \sqrt[3]{4}\cdot\sqrt[3]{2}$

$= 5\sqrt[3]{4} + \sqrt[3]{8}$ multiply radicands.

$= \boxed{5\sqrt[3]{4} + 2}$

(13) $(8-3\sqrt{2})(5+7\sqrt{2})$ multiply by FOIL

$\uparrow$ 1st term $\uparrow$ 2nd term $\uparrow$ 1st term $\uparrow$ 2nd term

$= 8\cdot 5 + 8\cdot 7\cdot\sqrt{2} - 3\sqrt{2}\cdot 5 - 3\sqrt{2}\cdot 7\sqrt{2}$

$= 40 + \underbrace{56\sqrt{2} - 15\sqrt{2}}_{\text{like radicals}} - 21\cdot\underbrace{\sqrt{2}\cdot\sqrt{2}}_{2}$

$= 40 + (56-15)\sqrt{2} - 42$

$= 40-42 + 41\sqrt{2}$

$= \boxed{-2+41\sqrt{2}}$

(14) $-36-5\sqrt{7}$

cannot be combined/simplified

(15) $(5\sqrt{7}+\sqrt{2})^2$

$= (5\sqrt{7}+\sqrt{2})(5\sqrt{7}+\sqrt{2})$ multiply by FOIL

$= 5\cdot 5\cdot\sqrt{7}\cdot\sqrt{7} + 5\cdot\sqrt{2}\cdot\sqrt{7} + 5\cdot\sqrt{2}\cdot\sqrt{7} + \sqrt{2}\cdot\sqrt{2}$

$= 25\cdot 7 + 5\sqrt{14} + 5\sqrt{14} + 2$

$= 175 + 10\sqrt{14} + 2 = \boxed{177 + 10\sqrt{14}}$

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16 $(8-\sqrt{5})(8+\sqrt{5})$ multiply by FOIL

$= 8\cdot 8 + 8\sqrt{5} - 8\sqrt{5} - \sqrt{5}\cdot\sqrt{5}$

$= 64 + (8-8)\sqrt{5} - 5$

$= 59 + 0\sqrt{5}$

$= \boxed{59}$

This is like a difference of squares — the middle terms add to zero.

We will capitalize on this result in 9.6.

Math 60 9.5 and 9.6 Formula in MathXL

In 9.5 and 9.6, we often multiply conjugates:

1. $(2+\sqrt{3})(2-\sqrt{3})$
2. $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$
3. $(3\sqrt{7}+2\sqrt{5})(3\sqrt{7}-2\sqrt{5})$

MathXL uses a formula: $(a-b)(a+b)=a^2-b^2$
or $(a+b)(a-b)=a^2-b^2$

to remind you that the FOIL process creates terms in the middle that add to 0 and disappear.

> This formula is just the factoring formula for the difference of squares $a^2-b^2=(a+b)(a-b)$, only it's written backwards.

Examples:

1. $$\underset{(a+b)(a-b)}{(2+\sqrt{3})(2-\sqrt{3})} = \underset{a^2}{4}\ \underbrace{-2\sqrt{3}+2\sqrt{3}}_{+0}\ \underset{-b^2}{-3} = \underset{\substack{a^2\\(2)^2}}{4} - \underset{\substack{b^2\\(\sqrt{3})^2}}{3} = \boxed{1}$$

2. $$\underset{(a-b)(a+b)}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \underset{a^2}{5}+\underbrace{\sqrt{10}-\sqrt{10}}_{+0}\ \underset{-b^2}{-2} = \underset{\substack{a^2\\(\sqrt{5})^2}}{5} - \underset{\substack{b^2\\(\sqrt{2})^2}}{2} = \boxed{3}$$

3. $$\underset{(a+b)(a-b)}{(3\sqrt{7}+2\sqrt{5})(3\sqrt{7}-2\sqrt{5})} = \underset{a^2}{9\cdot 7}\ \underbrace{-6\sqrt{35}+6\sqrt{35}}_{+0}\ \underset{-b^2}{-4\cdot 5}$$

$$= \underset{a^2}{63} - \underset{b^2}{20} = \boxed{43}$$